CHAPTER 4

Event: any collection of results or outcomes of a procedure.

Simple Event: an outcome or an event that cannot be further broken down into simpler components

Sample Space: for a procedure consists of all possible simple events; that is, the sample space consists of all outcomes that cannot be broken down any further

Notation for Probabilities:
P - denotes a probability.

A, *B*, and *C* - denote specific events.

P(*A*) - denotes the probability of event *A* occurring.

Basic Rules for Computing Probability:

- Rule 1: Relative Frequency Approximation of Probability

Conduct (or observe) a procedure, and count the number of times event *A* actually occurs. Based on these actual results.

$$P(A) = \frac{\text{\# of times } A \text{ occurred}}{\text{\# of times procedure was repeated}}$$

- Rule 2: Classical Approach to Probability (Requires Equally Likely Outcomes)

Assume that a given procedure has *n* different simple events and that each of those simple events has an equal chance of occurring. If event *A* can occur in *s* of these *n* ways, then

$$P(A) = \frac{s}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}}$$

- Rule 3: Subjective Probabilities: *P*(*A*), the probability of event *A*, is estimated by using knowledge of the relevant circumstances.

Law of Large Numbers: As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

Probability Limits: Always express a probability as a fraction or decimal number between 0 and 1.

- The probability of an impossible event is 0.
- The probability of an event that is certain to occur is 1.
- For any event *A*, the probability of *A* is between 0 and 1 inclusive. That is, $0 \le P(A) \le 1$.

Complementary Events: The complement of event *A*, denoted by *A*, consists of all outcomes in which the event *A* does not occur.

- $P(\grave{A}) = 1 - P(A)$

Odds:

- The actual odds against event A occurring are the ratio $P(A)/P(\bar{A})$, usually expressed in the form of a:b (or "a to b"), where a and b are integers having no common factors.

- The actual odds in favor of event A occurring are the ratio $P(\bar{A})/P(A)$, which is the reciprocal of the actual odds against the event. If the odds against A are a:b, then the odds in favor of A are b:a.
- The payoff odds against event A occurring are the ratio of the net profit (if you win) to the amount bet.
- payoff odds against event A = (net profit) : (amount bet)

Compound Event: any event combining 2 or more simple events.

Notation

$P(A$ or $B)$ = P (in a single trial, event A occurs or event B occurs or they both occur)

General Rule for a Compound Event:

find the total number of ways A can occur and the number of ways B can occur, but find that total in such a way that no outcome is counted more than once.

Formal Addition Rule OF Compound Event

$P(A$ or $B) = P(A) + P(B) - P(A$ and $B)$

- $P(A$ and $B)$ denotes the probability that A and B both occur at the same time as an outcome in a trial of a procedure.
- $P(A$ or $B)$, find the sum of the number of ways event A can occur and the number of ways event B can occur, adding in such a way that every outcome is counted only once
- $P(A$ or $B)$ is equal to that sum, divided by the total number of outcomes in the sample space.

Events A and B are disjoint (or mutually exclusive) if they cannot occur at the same time.
(That is, disjoint events do not overlap.)

Complementary Events: impossible for an event and its complement to occur at the same time.

- **$P(A)$ and $P(\bar{A})$, are disjoint**

Rule of Complementary Events:

$$P(A) + P(\bar{A}) = 1$$
$$P(\bar{A}) = 1 - P(A)$$
$$P(A) = 1 - P(\bar{A})$$

Notation: *P A* and *B*) = *P* (event *A* occurs in a first trial and event *B* occurs in a second trial)

Tree diagram: is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point.

Conditional Probability Important Principle: The probability for the second event *B* should take into account the fact that the first event *A* has already occurred.

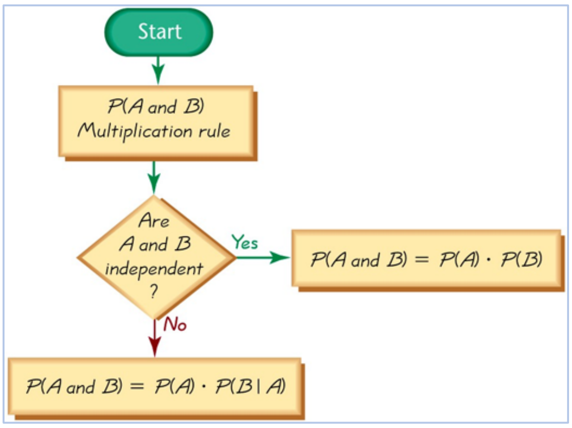
Notation: $P(B|A)$ represents the probability of event *B* occurring after it is assumed that event *A* has already occurred (read *B*|*A* as "*B* given *A*.")

Dependent and Independent:

- Two events *A* and *B* are independent if the occurrence of one does not affect the *probability* of the occurrence of the other.
- Two events are dependent if the occurrence of one of them affects the *probability* of the occurrence of the other, but this does not necessarily mean that one of the events is a *cause* of the other.

Formal Multiplication Rule: (Independent events)

- $P(A \text{ and } B) = P(A) \bullet P(B \mid A)$
- Note that if A and B are independent events, P (B A) is really the same as P(B).



(When applying the multiplication rule, always consider whether the events are independent or dependent, and adjust the calculations accordingly.)

"At least one": is equivalent to "one or more."

- The complement of getting at least one item of a particular type is that you get no items of that type.
- To find the probability of at least one of something, calculate the probability of none, then subtract that result from 1.

P (at least one) = 1 – P(none).

Conditional probability of *B* given *A* can be found by assuming that event *A* has occurred, and then calculating the probability that event *B* will occur.
$P(B|A)$ denotes the conditional probability of event *B* occurring, given that event *A* has already occurred, and it can be found by dividing the probability of events *A* and *B* both occurring by the probability of event *A*:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

To incorrectly believe that $P(A|B)$ and $P(B|A)$ are the same, or to incorrectly use one value for the other, is often called confusion of the inverse.

Simulation of a procedure is a process that behaves the same way as the procedure, so that similar results are produced.

random numbers are used in the simulation of naturally occurring events.

- A Table of random of digits
- STATDISK
- Minitab
- Excel
- TI-83/84 Plus calculator

Fundamental Counting Rule: For a sequence of two events in which the first event can occur *m* ways and the second event can occur *n* ways, the events together can occur a total of $m \cdot n$ ways.

Notation:

factorial symbol ! denotes the product of decreasing positive whole numbers.

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24.$$

By special definition, 0! = 1.

Factorial Rule: A collection of *n* different items can be arranged in order *n*! different ways.

Permutations:

- (when items are all different)

$$_nP_r = \frac{n!}{(n-r)!}$$

- (when some items are identical to others)

$$\frac{n!}{n_1! \, . \, n_2! \, .. \, . \, . \, . \, . \, . \, . \, . \, n_k!}$$

Combinations:

$$_nC_r = \frac{n!}{(n-r)! \, r!}$$

CHAPTER 5

Random variable a variable (typically represented by x) that has a single numerical value, (determined by chance), for each outcome of a procedure.

Probability distribution a description that gives the probability for each value of the random variable; often expressed in the format of a graph, table, or formula.

Discrete random variable either a (finite number of values or countable number of values), where "countable" refers to the fact that there might be infinitely many values, but they result from a counting process.

Continuous random variable (infinitely many values), and those values can be associated with measurements on a continuous scale without gaps or interruptions.

probability histogram is very similar to a relative frequency histogram, but the (vertical scale shows probabilities).

Requirements for Probability Distribution:

- $\sum P(X) = 1$ (where x assumes all possible values.)
- $0 \le P(x) \le 1$ (for every individual value of x.)

Mean, Variance and Standard Deviation of a Probability Distribution:

- $\mu = \sum[x \cdot p(x)]$ **(Mean)**
- $\sigma^2 = \sum[(x-\mu)^2 . P(x)]$ **(Variance)**
- $\sigma^2 = \sum[x^2 . P(x)] - \mu^2$ **(Variance (shortcut))**
- $\sigma = \sqrt{\sum[x^2 . P(x) - \mu^2}$ **(Standard Deviation)**

Round-off Rule for $\mu, \sigma \, and \, \sigma^2$: if the value of x are integers, round $\mu, \sigma \, and \, \sigma^2$ to one decimal place.

***Unusual* Results Range Rule of Thumb:** most values should lie within 2 standard deviations of the mean.

- Maximum usual value = $\mu + 2\sigma$
- Minimum usual value = $\mu - 2\sigma$

Unusual Results Probabilities: Using Probabilities to Determine When Results Are Unusual.

- Unusually high: x successes among n trials is an (unusually high) number of successes if P(x or more) ≤ 0.05.
- Unusually low: x successes among n trials is an (unusually low) number of successes if P(x or fewer) ≤ 0.05.

expected value of a discrete random variable is denoted by E, and it represents the mean value of the outcomes. ($E = \Sigma [x \bullet P(x)]$)

Binomial Probability Distribution requirements:

1. The procedure has a fixed number of trials.
2. The trials must be independent
3. Each trial must have all outcomes classified into two categories (commonly referred to as success and failure).

4. The probability of a success remains the same in all trials.

Notation for Binomial Probability Distributions:

S and F (success and failure) denote the two possible categories of all outcomes;
p and *q* will denote the probabilities of S and F, respectively, so

- $P(S) = p$ (p = probability of success)
- $P(F) = 1 - p = q$ (q = probability of failure)

n	denotes the fixed number of trials.
x	denotes a specific number of successes in *n* trials, so *x* can be any whole number between 0 and *n*, inclusive.
p	denotes the probability of success in *one* of the *n* trials.
q	denotes the probability of failure in one of the *n* trials.
P(x)	denotes the probability of getting exactly *x* successes among the *n* trials.

Important Hints:

- Be sure that *x* and *p* both refer to the same category being called a success.
- When sampling without replacement, consider events to be independent if $n < 0.05N$.

Three methods for Finding Probabilities:

- **Method 1: Using the Binomial Probability Formula**

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

for $x = 0, 1, 2, \ldots, n$

where

n = number of trials

x = number of successes among n trials

p = probability of success in any one trial

q = probability of failure in any one trial (q = 1 – p)

- **Method 2: Using Technology**

STATDISK, Minitab, Excel, SPSS, SAS and the TI-83/84 Plus calculator can be used to find binomial probabilities.

- **Method 3: Using Table A-1 in Appendix A** **(slide 31)**

Rationale for the Binomial Probability Formula:

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

$\frac{n!}{(n-x)!x!}$: Number of outcomes with exactly *x* successes among *n* trials

$p^x \cdot q^{n-x}$: The probability of *x* successes among *n* trials for any one particular order

Binomial Distribution: Formulas:

Mean $\mu = n \cdot p$
Variance $\sigma^2 = n \cdot p \cdot q$

Std. Dev. $\sigma = \sqrt{n \cdot p \cdot q}$

Where
n = number of fixed trials.
p = probability of success in one of the n trials.
q = probability of failure in one of the n trials.

Poisson Distribution: is a discrete probability distribution that applies to occurrences of some event over a specified interval. The interval can be time, distance, area, volume, or some similar unit

Formula

$$P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!} \quad \text{where } e \approx 2.71828$$

Requirements of the Poisson Distribution:

- The random variable x is the number of occurrences of an event over some interval.
- The occurrences must be random.
- The occurrences must be independent of each other.
- The occurrences must be uniformly distributed over the interval being used.

Parameters :

- The mean is μ.
- The standard deviation is $\sigma = \sqrt{\mu}$.

The Poisson distribution differs from the binomial distribution in these fundamental ways:

• **The binomial distribution is affected by the sample size n and the probability p**	• **The Poisson distribution is affected only by the mean μ**
• **The binomial distribution the possible values of the random variable x are 0, 1, . . . n**	• The Poisson distribution has possible x values of 0, 1, 2, . . . , with no upper limit.

The Poisson distribution is sometimes used to approximate the binomial distribution when n is large and p is small.

If both of the following requirements are met,

- $n \geq 100$
- $np \leq 10$

then use the following formula to calculate μ,

Value for µ

- $\mu = n \cdot p$